

Simple Representations of Complex Networks: Strategies for Visualizing Network Structure*

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Abstract

Despite a growing number of powerful techniques for analyzing networks, network researchers often find themselves in need of simple and efficient ways to communicate their results to a nonprofessional audience.

How can we visualize network structures in a simple manner while controlling the location of errors which may result from such simplifications ?

We propose a family of strategies, that includes at least three different steps:

1. the choice of simple geometric shapes as a priori constraints to limit the permissible spatial locations of network nodes
2. the choice of a specific structure characteristic as a criterion for optimization
3. a parsimonious algorithm to optimize the spatial distances between the elements of various sets contained in the network data.

Starting with simple examples of social structures, we will demonstrate that such strategies work well. Examples from 'real world data' will show that this is a practical way to handle even quite complex data.

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More information about network visualization is available via Mosaic under following URL:
<http://www.mpi-fg-koeln.mpg.de/~lk/netvis.html>

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Despite a growing number of powerful techniques for analyzing networks, network researchers often find themselves in need of simple and efficient ways to communicate results to a non-professional audience.

From the beginning, network analysis has been accompanied by more or less accurate attempts to visualize structure, mostly grounded on an intuitive understanding. There are many examples in the literature where simple geometric shapes (lines and circles) have been used to illustrate the underlying structure of datasets.

What we want to do is follow this tradition *but in a systematic and automatic way* by proposing a family of algorithms which try to grasp the core of the underlying structure. If they do exactly this, they are cheap in terms of computational demands.

How can we visualize network structures in a simple, systematic and parsimonious manner and check at the same time for errors which may result from such simplifications ?

After some general remarks about simplification and its problems from the perspective of model building, we will review different types of information that can be used to visualize network structures and propose how such information can be fitted into a constrained model space and how to handle errors.

A second part of this paper will analyze how the proposed procedure works for handmade toy examples containing simple structures and then proceed to compare results from the proposed algorithm with those of an extensively studied dataset.

Finally, we will apply the algorithm to complex 'real world datasets' and demonstrate its usefulness, when introducing a priori designs of the solution space.

1 Simplification

Many statistical procedures try to get most of our data (and often provide us with more information than we actually need to answer our scientific hypotheses). A common strategy is to reduce the amount of information by fitting it into more simple concepts: clouds of points are represented by best fitting lines, complex similarity structures are represented in the form of binary trees.

The strategy we follow in this paper is to constrain the solutions to simple patterns, patterns which are known to put relatively little demand on the perceptual skills of an audience.

This should not be misunderstood as an argument for replacing more elaborated, statistically well grounded procedures, but for supplementing them with a family of 'low cost' tools. Such tools serve their purpose when they are able to grasp the most important features of the underlying information.

A general expectation from the perspective of model building is that the more simple a model and the more restricted the set of its solutions, the likelier we are to end up with a very poor fit of the empirical data. Of course this will depend on our data.

On the other hand even a poor model can be useful to some degree, depending on whether it still reflects the most important features of the underlying structure and misplaces only less important elements. This assumes an algorithm which explicitly takes some criteria of structure into account and focuses on solving the fit for the core of the structure first, proceeding to less important elements at a later stage.

1.1 Model

We have observed that circles have been widely used in the literature to illustrate network results. There is at least one formal argument which helps us to understand why. If the elements of a set of nodes are equally spaced onto a circle, the sum of distances from each node to all other nodes is equal.

This seems to be a particularly favorable way to illustrate connections in a network. It allows even naive observers to perceive the contrast between a specific solution and the idealized pattern from which it deviates. (as an easily perceivable null hypothesis). This would also mean that other shapes could be used for specific problems as long as they offer regularity, like triangles or squares.

Constraining the possible solution to a set of locations being a member of a specific shape specifies at the same time a distance matrix between all permissible positions of the model.

If p is the number of available positions in the solution space, the simple model can be described by the distances between all permissible positions, a matrix $\mathbf{D}_{[p,p]}^*$.

1.2 Data

Typical network data describe one or two sets of nodes and the relations among, respectively between the nodes. The data do not necessarily contain, however, any information about the nodes themselves.

A typical problem which is solved by many network techniques is to classify the nodes on the basis on their direct or indirect links or even more complex information such as similarities or distances.

Since an adjacency matrix is a matrix of 1-step distances, we can interpret it as a distance matrix $\mathbf{D}_{[m,n]}$ describing the distances between two sets of nodes, where m is the number of nodes in the row set and n the number of nodes in the column set. All directly connected pairs have an entry of one and all unconnected pairs an entry of zero, indicating the absence of a relation.

There is no reason not to use other distance matrices, which usually contain richer information than the adjacency matrix of a graph: the reachability matrix as a graph-theoretic distance measure or even similarities or distances resulting from more complex operations.

Roger Shepard's (1972) taxonomy of principal datatypes as a basis for multidimensional scaling procedures provides the concepts necessary to distinguish the basic information from technical transformations applied to yield an empirical distance matrix, which allows a fitting algorithm to find an optimal solution for a given model space.

2 Optimization

2.1 Measures for Structure

The choice of a measure for structure depends on what is to be conveyed to a particular audience. The algorithm will try to do its best to fit the empirical distances \mathbf{D} into the space of the permissible target locations of the model which is described by \mathbf{D}^* .

There is a large number of centrality concepts readily available, which describe with statistics what people intuitively look for when they visually inspect graphical representations of network data.

A simple concept is Freeman's *degree of centrality*, based on the rows of the adjacency matrix. His concept of *closeness*, which evaluates centrality on the basis of the matrix of geodesics, is more informative, additionally taking the shortest indirect distances into account (Freeman (1979)).

Regarding measures of centrality there is a particularly intense discussion of how these indices relate to the concept 'social power', and there exist advanced formal concepts how to approach this theoretical problem methodologically. (Bonnacich (1987) or Friedkin (1991)).

2.2 Controlling the Location of Errors

In a simple case the model space D^* allows for as many positions in two-dimensional space as there are nodes in your data. Most likely under the given constraints this will not make it possible for each node to take a position where its overall distance in the model space (to all nodes to which it is linked directly) is minimal.

While this is the basic problem of trading simplicity for exactness, we are willing to sacrifice exactness to a certain degree if we force the solution into a constrained space. A pragmatic solution to this problem is to try to get as much exactness as possible under the given circumstances: for the visual inspection it is often sufficient if the most important elements are placed best, i.e. are located in the most central positions available in a model space.

This also means that we are willing to accept errors if they occur to structurally less important elements: if not possible we are willing to have them misplaced to a certain degree.

The general idea of controlling where the errors are placed in the model space means using the optimization criterion itself to specify priorities for misspecification: if the structurally most important elements are placed first, less important elements are forced to the remaining positions under more restricted alternatives for positioning.

Depending on the quality of your data, such a strategy might save you from interpreting white noise contained in your data, if you expect noise to affect mainly the periphery of the underlying structure.

2.3 An Algorithm

The general idea of the proposed algorithm is to place the structurally most important elements first and not to allow less important elements to change the locations assigned to more important elements. This reduces the amount of computations necessary to a considerable degree. While this works perfectly for data where no two nodes have the same structural importance, i.e. where there is a lot of structure in the empirical data, care has to be taken for less structured data: when there are ties in the rank orders of importance, it means that there are nodes that cannot be distinguished from each other.

1. sort the all nodes of the graph according to their structural importance (your choice of criterion to represent structure)
2. for all elements from most structurally important to least important do
3. evaluate all admissible (not marked) locations in the model space for their minimal overall distance to those nodes to which there is a link
4. assign the node under consideration to a position in the model space which has not been assigned (marked) yet and for which the sum of distances is minimal
5. mark the position assigned as being taken
6. repeat step (2-5) until all elements have been assigned
7. unmark all nodes and repeat step(2-6) several times

In these cases the implementation of the algorithm should take care to assign the priority sequence randomly for each set of indistinguishable nodes for different iteration steps. If there is little or almost no structure in the data, the advantage in computing demands is entirely lost: one has no choice there but to evaluate all possible permutations of locations for an optimal solution.

Finally it should be pointed out that the algorithm uses only rankorders of the overall distances in the model space, which makes it quite robust to different designs of the solution space as long as these designs provide enough variation in the distances among all target locations.

3 Examples

3.1 Toy Examples

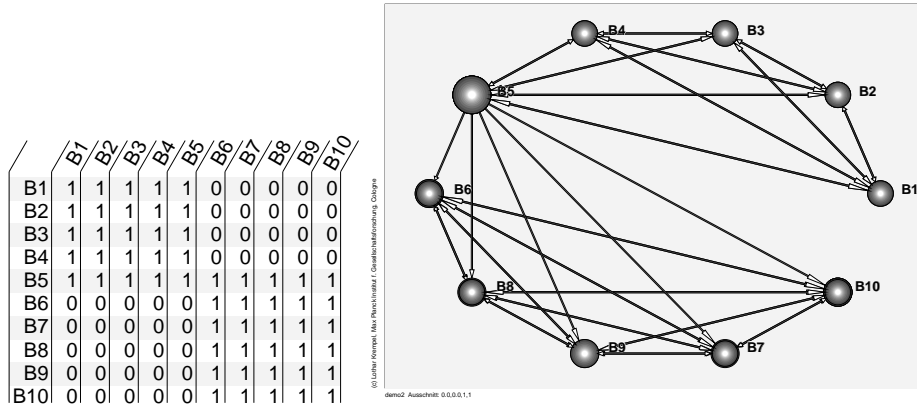


Figure 1: Toy examples: two cliques and a star

The first feature we would like to illustrate is that the proposed algorithm is able to visualize the underlying structure of 'handmade' examples containing idealized structure only.

Figures 1 and 2 provide the solutions for two such toy examples. The first example contains two cliques which are linked by a star {B5}, having direct connections to all other nodes in the system. The second example again contains two cliques, but it differs from the first in that there is one node in each of these cliques linked to the other clique. Nodes {B5} and {B6} bridge the gap between the otherwise unconnected subsystems {B1, B2, B3, B4, B5} and {B6, B7, B8, B9, B10}.

In both examples we find that the cliques are placed contiguously on the circle of the solution space, while the elements, linking the two subsystems are placed in between.

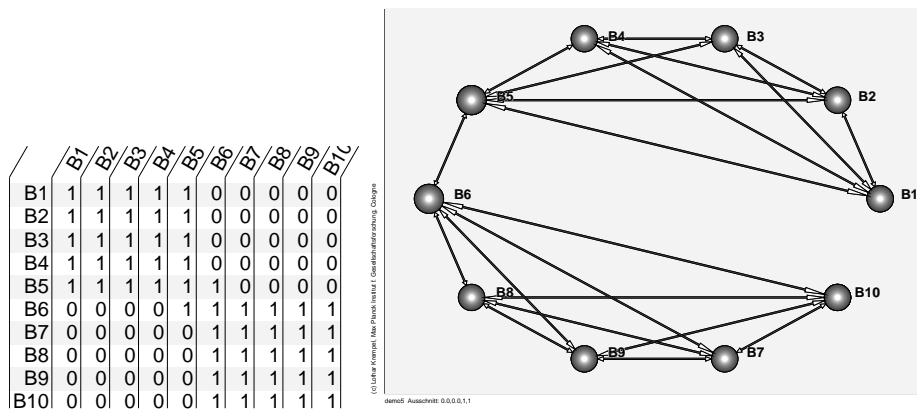


Figure 2: Toy examples: two cliques and a bridge

3.2 A Well-Known Example

A more complex and well-studied dataset is that of Doreian (1989), which describes the structure between politicians engaged in decision-making in a county in the American Midwest. This dataset has been extensively studied using various analytic procedures (Doreian (1988), (1989), Doreian and Albert (1989)), so that the scope and the subtleties of its solutions are quite well known. A reanalysis with our algorithm is therefore a kind of validation for our procedure. The comparison with the outcomes of various other analytical procedures also helps us to understand, how our algorithm works.

Doreian's network is made up of the fourteen most prominent political actors of a Midwestern County engaged in making decisions about the construction of a new jail. Seven of these make up the County Council which is the legislative and taxing authority of the county. Members serve for four years, with one of them as the *Council President*, the other six being labeled *Council 1* through *Council 6*. Two of these lost their reelection bids: the *Former Council President* and the *Former Co President*. Other actors are the *County Executive* as the chief officer of the county, the *County Auditor* responsible for the county's administration, the elected *Sheriff* as the law enforcement officer and the *County Prosecutor*, the chief legal officer of the county.

Doreian and Albert (1986) hypothesised that the political actors would be partitionable into two camps. One would be associated with the *County Executive*, while the other would be associated with the *County Auditor*. Their second hypothesis was that the distribution of votes would be conditioned by the partitioned network of strong ties among the political actors. Both hypotheses were confirmed, leading them to conclude that the structure of the network of strong political ties was responsible for the prolonged inaction of the County Council.

.. on the right the diagram is a clustered set of five points containing the County Auditor (B), while the left contains a clustered set of seven points containing the County Executor (A). ... In the County Auditor's alliance, Council 5 (I) and Council 6 (J) occupy the same location. The Council President H is located close to B and the City Mayor (M) can be grouped with the other four points. In the left hand cluster, the Sheriff (C) and Council 4 (G) are close, as are Council 1 (D) and Council 3 (F). The County Auditor (A) can be grouped with C, G, D and F. Finally Council 2 (E) and the County Prosecutor (N) join the cluster but at a greater distance. At the center of the Euclidian space is the Former Council President (L) with the Former Council member (K) at the periphery of the diagram. p. 285 f.

In Figure 3.2 we present our solution for Doreian's political actors¹, yielded by applying our algorithm to the solution space of 14 equally spaced locations on a circle, which corresponds to the number of actors in the network.

We used the *degree of centrality* to specify the priority sequence of how nodes enter into the computations. This makes possible errors likely to occur only for peripheral actors.

There are the same two cliques as identified by Doreian: the A clique { A, C, D, E, F, G, N } and the B clique { B, H, I, J, M }. The *Former Co President* { L }, as the most central actor in the network, is located between the two subsystems A and B, with the *Former Council* {K} attached to {L}.

¹The size of the nodes in this and all following figures is proportional to the sum of both the in- and outdegree of a node. This characterizes a node by all direct activities it is engaged in and reflects the direct component of centrality.

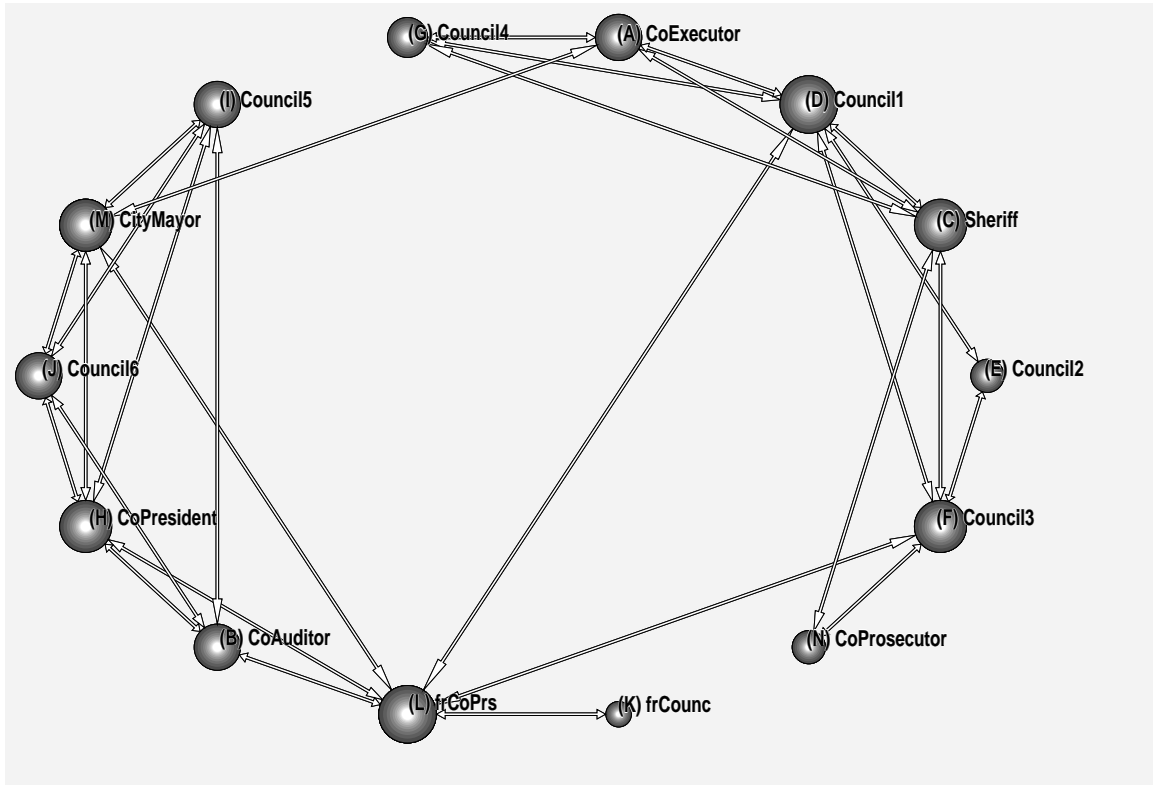


Figure 3: Doreian politicians: solution based on direct links only

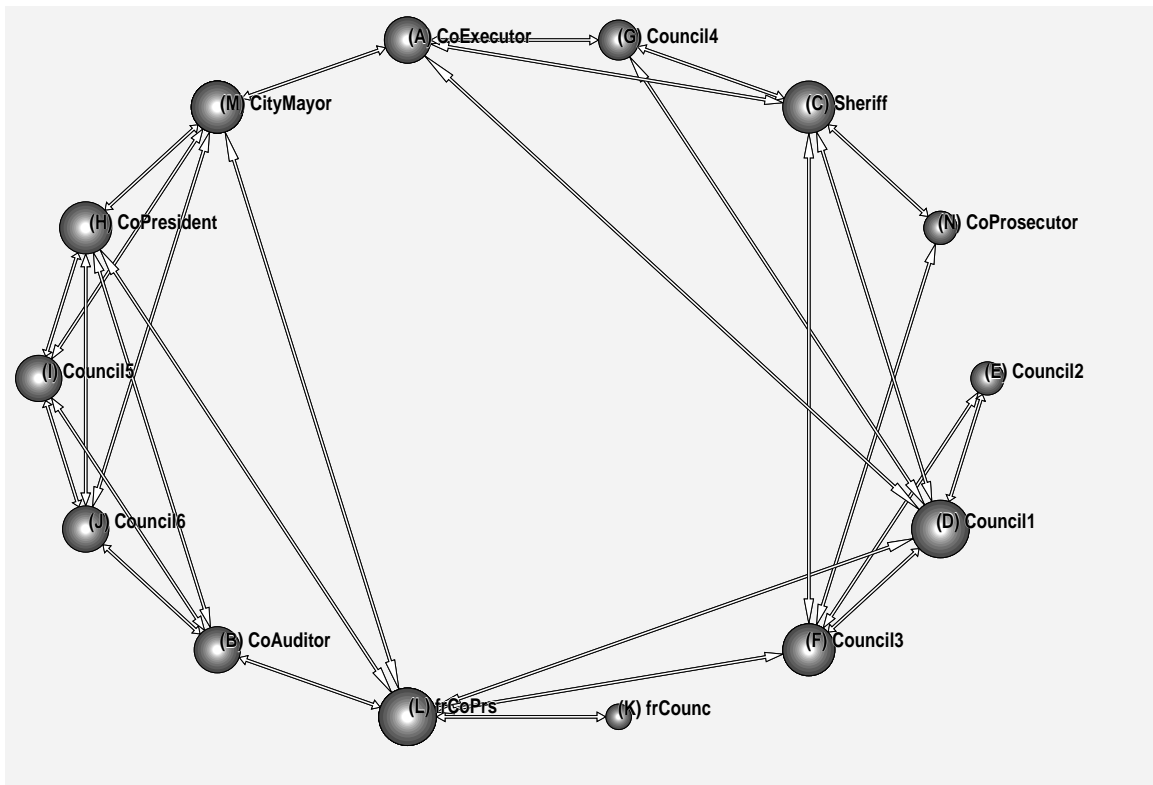


Figure 4: Doreian politicians: solution based on geodesics

What is different, however, in Figure 3.2 is the ordering of nodes in each of these cliques. This is not completely surprising, since Doreian computed his MDS solution on the basis of geodesics (using the shortest graph-theoretic distances between all pairs), while our solution is based on the direct links only. In our case, it is evident that the boundary spanning actors {M,H} and {A, D, F} in each of the cliques are located closer to the center of the subsystems, while completely peripheral actors 'buried with the alliances' {G, N, E, K}, are found on the subsystem's periphery, with exception of {E}.

A further test of our algorithm is to use the same graphtheoretic distance information that was used by Doreian. As this information accounts additionally for the shortest indirect links between any actors, it will raise those actors in the rank-order of centrality (closeness), who occupy boundary spanning positions.²

If we use the rank-order in 'closeness' to specify the priorities in which nodes are taken into account, 'boundary spanners' should move towards the intersection of both subsystems. The results of this enhanced solution are shown in Figure 3.

If we view the ordering in the second solution from the position of {L}, {B,H,J,M,I} is changed to {B,H,J,I,M} whereas {N,F,E,C,D,A,G} is changed to {D,E,F,N,G,C,A}. With the exception of {F}, who is located in between {D} and {A}, the results of this analysis have become similar to Doreian's MDS solution and mimic even the internal organization of Doreian's MDS clusters to some degree.

The primary focus in this section has been to demonstrate that the algorithm yields acceptable results when compared to more powerful analytic procedures. We have shown, that our algorithm is able to find cliques or clusters of cohesion. We have demonstrated that it can use quite extensive information, and, that the results in such cases mimic the results obtained with more powerful analytic procedures, despite the given constraints of a strongly restricted solution space.

We have argued that the real virtue of the proposed procedure is that the algorithm works on an a priori constrained solution space, giving us the means to control the complexity and the design of a solution. While this may not have become evident in this section, we will try to demonstrate exactly this in the next section, when we use the proposed strategy to visualize more complex datasets and use the outlined procedure as a building block for more complex designs of the solution space.

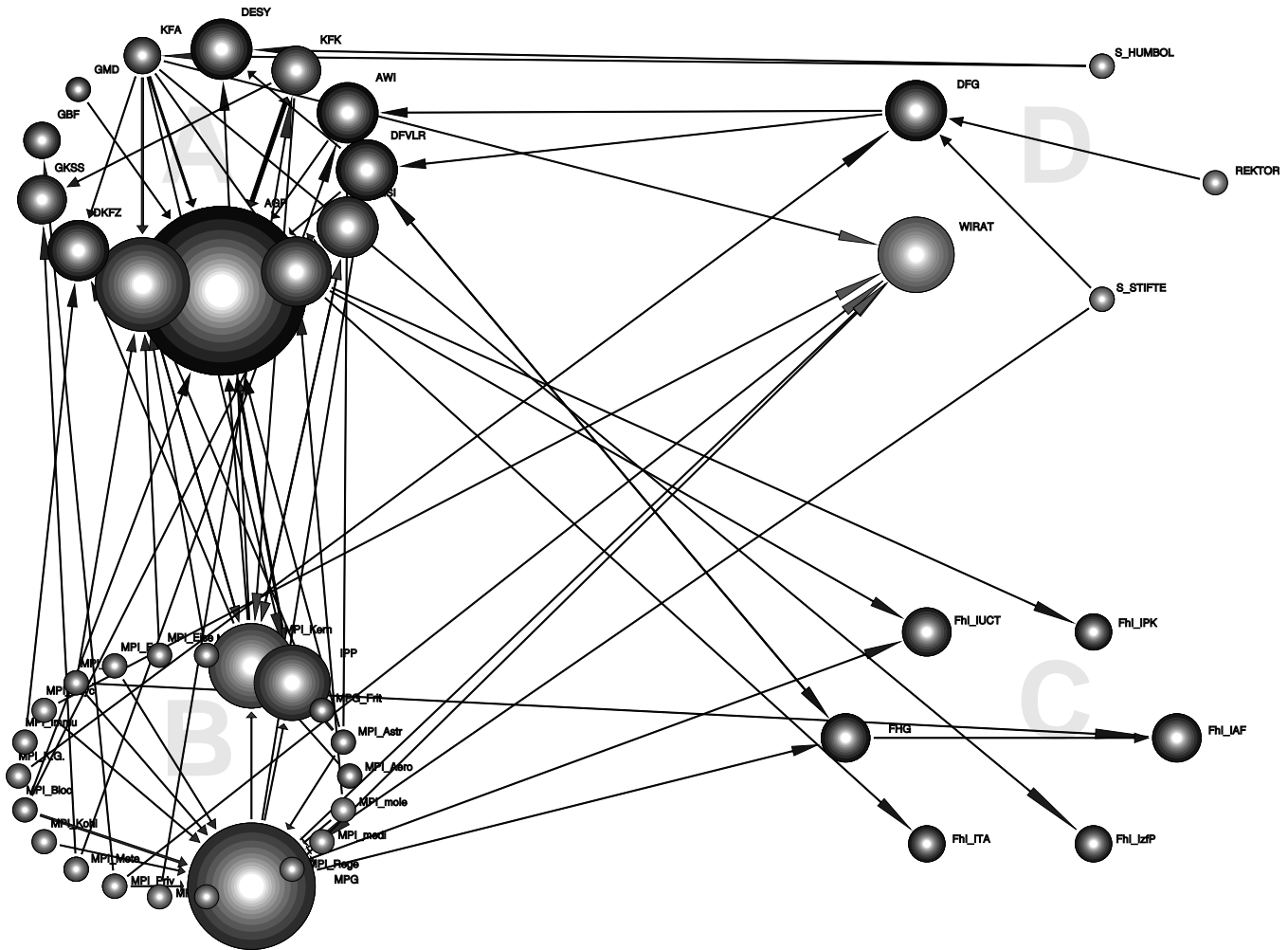
²The two different rank-orders of centrality are listed in Table 8: for direct links the algorithm proceeds according to the rank-order given in the column 'Degree of Centrality', while for geodesics the rank-order given under 'Closeness' is used. As the reader can verify, the 'boundary spanners' have higher ranks (are more central) when geodesics are used.

3.3 Complex Examples

The case of interest here is a system of three organizational systems for which a complete matrix of interlocks between their advisory bodies has been reconstructed on the basis of available documents.

The specific missions of the subsystems as they are laid down in their organizational charters are basic research, implementation of large-scale projects and applied research. There is an on-going political discussion about whether the overall design facilitates an efficient transfer of basic knowledge to more applied levels and industry.

are numerous opinions on how and whether such a system can be adequately governed and its overall efficiency enhanced, and while these opinions draw on an almost infinite number of



A system of three Research Organizations (A,B,C) and their relations to various committees and organizations giving recommendations for science policy (upper right circle) (D)

Figure 5: A System of Organizational Systems

causes and explanations, knowledge about the system's actual structure and its actual patterns of coordination are sparse and hidden in many dispersed documents.

Without our going into detail, it seems worthwhile to simply describe the system as a whole on the basis of the more or less institutionalized relations in and between its components. This will eventually allow us to locate coherent boundary spanning subsystems. Such a finding could prove quite useful for the organizations when they want to articulate their situation vis-a-vis one of several commissions which are installed for setting priorities for the entire system and for redistributing money between the subsystems.

The network data of the total system can be expected to reflect part of the inner structure of each of the systems, while the intersystem domain can be expected to reflect partly relations between the top levels of the subsystems and partly the disciplinary coordination between members of different subsystems working in similar substantive domains.

Choosing among the various alternatives for visualizing this dataset, we have decided not to start from the systems perspective and decompose the total system using the empirical network information, but to contrast the empirical data with the more commonly understood institutional layout as it corresponds to the formal charters of the subsystems.

This has led us to assign each of the subsystems to different but equal-sized shapes, based on the common a priori understanding and formalized design. The shapes themselves are organized to form a regular pattern. By doing this, we have constrained the solution for the total system in various ways: a member of subsystem A may only be placed on one of the permissible locations of shape A, the same holding true for members of the other a priori sets B, C and D.

The algorithm itself exploits the full matrix describing the entire system. In order to optimize for the degree of centrality a member of A, B .. C has in the entire system, it tries to place the most central elements first. The centrality of each actor reflects its embeddedness into the local subsystem as well as its links into the global system. The solution will show that position in the target space which minimizes both the internal and external relations ³.

The result is found in Figure 5. It exhibits a considerable amount of ordered information: nodes with no intersystem orientation are moved to the nonadjacent locations of each shape: the entire system's periphery. Players in the intersystem domain are oriented toward their main partners in the other shapes. Furthermore, organizations with access to more than one of the neighbouring systems are placed in between.

While we find the overall result to be more impressive than we had hoped for, and while it has proven to be useful for our purposes, Figure 5 also exhibits areas in which the placement of elements has not been carried out in such a way that the location of each single element is always best: an indicator of a too-rigorously constrained solution space. Depending on the degree of outward orientation in the subsystems, it happens that organizations are moved towards the periphery despite being players in the intersystem domain. These misplacements are errors that result from a solution space which does not provide enough positions on the adjacent side of the shape. Nevertheless, even these errors are controlled to the degree that such elements differ in their (ranks) of centrality, which the proposed algorithm makes up for. ⁴

³To prevent unwanted weights from entering into the optimization procedure, the algorithm locates each of the organizational shapes on a second circle, choosing the diameters of the larger circle to correspond to the diameters of the shapes representing the single systems.

⁴While Figure 5 was one of the first results we achieved with our algorithm, there is also some further criticism because circles C and D seem to be interchanged. This results from using a square arrangement for the four circles.

The results of such a modification can be seen in the next example. Figure 6 tries to visualize the three organizational systems of the previous example and their interface with the 25 most important industrial partners (having more than one link into the system). These industrial partners can be expected to have some effect on the coordination which occurs in the total system's outcome.

The number of (equally spaced) positions available on each shape has been increased by 50 percent, while in the previous example there were exactly as many positions in the solution space as there were elements in each of the systems.

As we have no information on the dependencies among the industrial actors (the relations available are only the relations to each of the three organizational systems (a rectangular matrix), we have chosen a larger circle to constrain the placement of the industrial partners, surrounding the internal system.

As Figure 6 shows, all of the industrial partners crowd the southern hemisphere of the outer circle, while the north is unpopulated: all industries linked to the inner top circle B have also ties with either A or C.

At the same time, the placement of the organizations in the inner circles is very informative: while the members in the inner top circle B crowd the southern locations only, most of the elements in the lower right circle A have moved north: both (A and B) seem to build the *inner core* of the total system, while almost all members of C, the lower left circle, have moved west i.e. are almost exclusively linked to industrial partners.

Interesting phenomena can be detected for A and C, the two lower circles of the inner system. There are three nodes in the north east region of C which are connected to the inner core, while the top-level organization of C takes an intermediate location north-northwest: an appropriate place for organizations having a balanced ratio of ties to the inner core as well as to industrial organizations. For A, the lower right circle, we find only one single node to be placed in the south east section, indicating that this organization is linked to more industrial partners than to research organizations.

Focusing on the western sphere of A, we find a large node (having many ties) in the south west *without* direct links to the adjacent members of C. In this case, the short intersystem distance in the model space is not based on direct access but on the *structural equivalence* of the south east of C and west of A: nodes are placed close to each other when there is an overlap in their ties to third partners (industrial actors holding seats in both organizational systems) even if there are no direct links.

Looking at the most important industrial actors, whose importance can be read from the size of their nodes (symbolizing the number of links they have into the inner systems), we find the important ones to have access to the inner core. Without our a priori constraints of the the region of minimal distances for these would have been somewhere in between the inner circles, the adequate location for potential intermediaries between the inner circles. On the other hand, our constrained solution contains interesting information, too. As the distance to the center of the entire system is equal from all locations of the outer circle, the 'best' of the available locations for industrial actors which have access to the center is the one in which they can get closest to most of the research organizations in which they hold seats. The actual solution is therefore informative regarding the main orientation of these actors.

Characterizing the total system on the basis of its ties among boards of directors and advisory committees, we find an inner core which is almost exclusively formed by members of A and C (the upper and lower right circles) with only few intermediaries of B (the lower left system) attached.

While this could indicate that there is too little transfer into the lower left system, we observe at the same time that the most important industrial actors which we have forced by design to the outer periphery, have links to the inner core, and are thus at the same time intermediaries between the inner core and the lower left.

4 Conclusions

Throughout this paper we have followed the idea of enhancing communication of network results by approaching the complexity of visualizations. We have proposed an algorithm and shown that it finds cliques and clusters of cohesion for simple structures and also for more complex datasets.

Due to the fact that this algorithm uses a precedence criterion to attach errors to structurally less important elements, it gives us the opportunity to work with highly restricted solution spaces, while ensuring that the visualizations grasp the most important parts of the underlying structure.

The formal procedure itself can be applied to any distance-type information derived from network data. These data may differ in the richness of information they provide, moving the focus of visualization from direct links only to the possible consequences of indirect links. Therefore the road to more theory-guided centrality measures in the future is open.

Moving to large and more complex networks, we have demonstrated that the outlined procedure can be used as a building block to approach networks of networks: we have used additional a priori information to assign nodes to several shapes. The algorithm itself can handle the information even in these cases. It tries to use the empirical information on the basis of the a priori constraints to fit the data.

As a priori designs affect the distances in the model space and thus the evaluation of the minimal positions, more knowledge and experience is needed to fully exploit the range of alternative designs.

Visualizations need not to be perfect in terms of fit but fulfill their purpose when they guide the audience to the most important characteristics of the underlying subject. Nevertheless thinking in terms of a sequential approach from simple illustrations to a parsimonious model, we would benefit from more guidance on how to find the ideal balance between simplicity and fit. As this point we think of rank correlations between the model and empirical distances, but it is still unclear whether they can serve to sort out the best means by which we can expect to find this balance: by increasing the degrees of freedom or by using alternative designs. If we had such criteria, they could also help us to choose among a variety of alternative designs.

A Data

	A-CoExecu	B-CoAudito	C-Sheriff	D-Council1	E-Council2	F-Council3	G-Council4	H-CoPresid	I-Council5	J-Council6	K-Fr Counc	L-FrCoPres	M-CityMayo	N-CoProsec
A-CoExecu			1	1			1						1	
B-CoAudito								1	1	1		1		
C-Sheriff	1			1	1	1								1
D-Council1	1		1		1	1	1					1		
E-Council2				1		1								
F-Council3			1	1	1							1		1
G-Council4	1		1	1										
H-CoPresid		1							1	1		1	1	
I-Council5		1						1		1				1
J-Council6		1						1	1					1
K-Fr Counc												1		
L-FrCoPres		1		1		1		1			1		1	
M-CityMayo	1							1	1	1		1		
N-CoProsec			1			1								

Figure 7: Political Actors (Doreian (1988))

	Degree	Degree	Geodesics	Geodesics
Rank1	D councl1	(1) 6	L FrCoPrs	(1) 22
Rank2	L FrCoPrs	(1) 6	D councl1	(2) 24
Rank3	H coPresd	(2) 5	F councl3	(3) 25
Rank4	C sheriff	(2) 5	M cityMay	(3) 25
Rank5	M cityMay	(2) 5	A coExecu	(4) 26
Rank6	F councl3	(2) 5	H coPresd	(5) 27
Rank7	A coExecu	(3) 4	C sheriff	(6) 28
Rank8	B coAudit	(3) 4	B coAudit	(7) 29
Rank9	J councl6	(3) 4	G councl4	(8) 30
Rank10	I councl5	(3) 4	I councl5	(9) 33
Rank11	G councl4	(4) 3	J councl6	(9) 33
Rank12	N coProsc	(5) 2	K frCounc	(10) 34
Rank13	E councl2	(5) 2	E councl2	(10) 34
Rank14	K frCounc	(6) 1	N coProsc	(10) 34

Figure 8: Degree of Centrality and Closeness (geodesics)

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